Trigonometry

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Book: *Trigonometry 3rd ed.* by Dennis Zill, © 2012 *Jones & Barlett*, ISBN 978-1-4496-0604-6

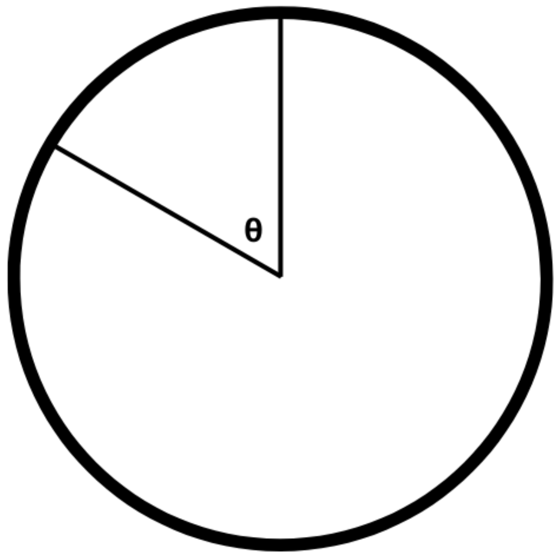
Topic	Problem 1	Problem 2	Problem 3
1. Right triangle trigonometry			
1.1 Angles and their measure	✓		
1.2 Right triangle trigonometry	✓	✓	✓
Quotient identities	✓	✓	✓
Reciprocal identities			
Cofunctions	✓	✓	
Some pythagorean identities	✓		
1.3 Trig functions of special angles	✓		
30, 60, 90			
45, 45	✓		
Minutes and seconds	✓		
1.4 Trig functions of general angles			
Reference angles			
2. Unit Circle			
2.1 Describing the unit circle	✓		
2.2 Circular functions			
Unit circle visual aid			
Domain and range	✓	✓	
Odd and even functions			
$\sin(x \pm \pi), \cos(x \pm \pi)$	✓	✓	✓
$\sin(\pi - x), \cos(\pi - x)$	✓	✓	✓
2.3 Graphs of sine and cosine functions			
$\sin(x)$, $\cos(x)$	✓	✓	
$\sin(x + \alpha)$, $\cos(x + \alpha)$	✓		
$\sin(x - \eta)$, $\cos(x - \eta)$	✓	✓	
2.4 Graphs of other trig functions			
$\tan(x)$, $\csc(x)$, $\sec(x)$, $\cot(x)$			
$\tan(x \pm \lambda)$, $\csc(x \pm \lambda)$, $\sec(x \pm \lambda)$, $\cot(x \pm \lambda)$			
2.5 Special identities	✓		
Half-angle identities	✓		
Trigonometric substitution	✓		
2.6 Inverse trig functions	✓		
2.7 Trigonometric equations			

3. Applications of trigonometry			
3.1 Solving right triangles			
3.2 Applications of right triangles			
3.3 Law of sines	✓		
3.4 Law of cosines	✓		
3.5 Simple harmonic motion			
Topic	Problem 1	Problem 2	Problem 3
4. Complex numbers			
Preliminary to complex numbers			
4.1 Complex numbers	✓	✓	
4.2 Complex solutions			
4.3 Trigonometric form of complex numbers			
4.4 Euler's formula	✓	✓	✓
4.5 Exponential form of complex numbers			
4.6 DeMoivre's theorem	✓		
4.7 Roots of complex numbers	✓	✓	
5. Exponential and logarithmic functions			
5.1 Exponential functions			
Transformations of exponential functions			
5.2 Logarithmic functions	✓	✓	
5.3 Exponential and logarithmic equations			
5.4 Exponential and logarithmic models			
5.5 Hyperbolic functions			
6. Analytic geometry			
6.1 Lines	✓		
6.2 Conics	✓		
6.3 Parabolas			
6.4 Ellipses			
6.5 Hyperbolas			
6.6 Rotation of axes	✓	✓	
6.7 Parametric equations			
7. Polar Coordinates			
7.1 Polar coordinates	✓	✓	
7.2 Graphs of polar equations	✓		
7.3 Conic sections in polar coordinates			

Problems

1.1 Angles and their measure

1. Convert 1° to radians
2. Convert 1 radian to degrees
3. Use the diagram to show the definition of θ

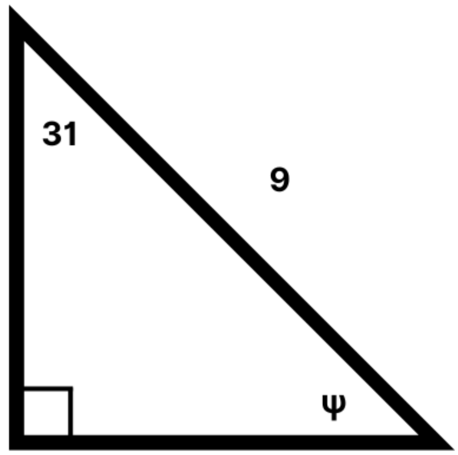


4. What happens when the arclength $= \frac{\pi}{2}$?

1.2 Right triangle trigonometry

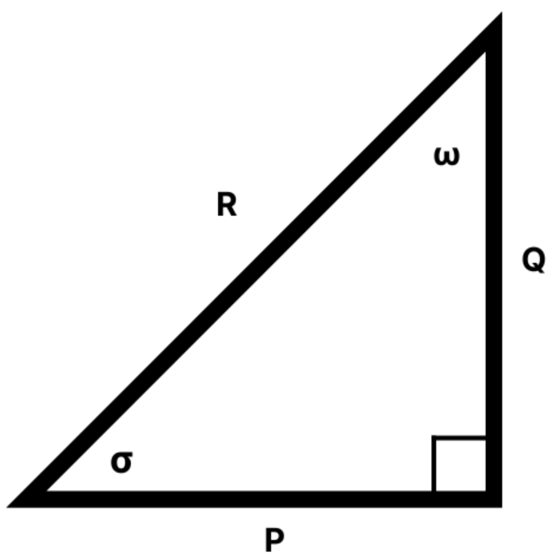
1. Given $\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$, find the value of the remaining four trig functions.

2. Find all values of the six trigonometric functions for ψ in the triangle.



3. Given that $\cos\rho = \frac{1}{3}$ and $\tan\rho = 2\sqrt{2}$, find $\sin\rho$.

4. Using the diagram, list the six trig identities with their appropriate cofunction.



5. What do you notice about $\sigma + \omega$? Find ω in terms of σ.

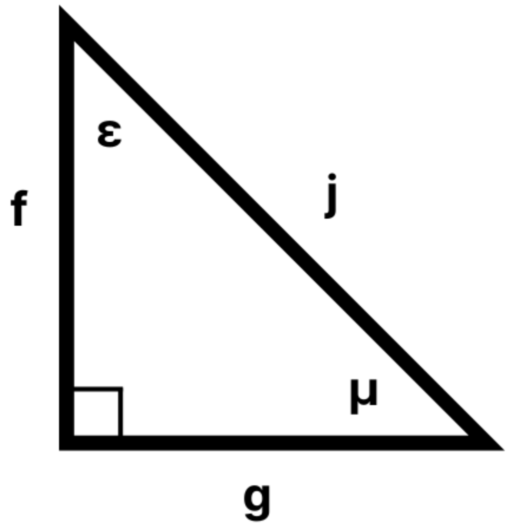
6. Relist the cofunctions in terms of σ.

7. $\cos\frac{\pi}{4}$ is what cofunction? List steps.

8. $\cot\frac{\pi}{6}$ is what cofunction? List steps.

9. $\tan 15^{\circ}=$

10. csc 27°=



11. Use the triangle and the pythagorean theorem to derive the trig identities...

$\sin^2 \epsilon + \cos^2 \epsilon = 1$

$1 + \tan^2 \epsilon = \sec^2 \epsilon$

$\cot^2 \epsilon + 1 = \csc^2 \epsilon$

12. If μ is an acute angle, and $\tan \mu = \sqrt{7}$, find $\cos \mu$. Rationalize any denominators.

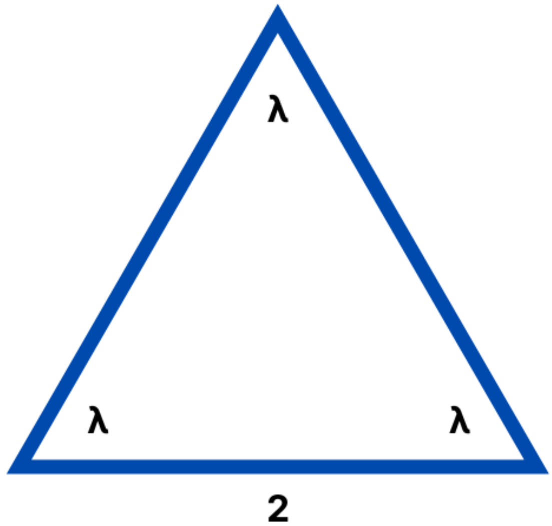
13. Find the exact value. Do not use a calculator.

$$3 \sin^2 \frac{\pi}{9} + 3 \cos^2 \frac{\pi}{9}$$

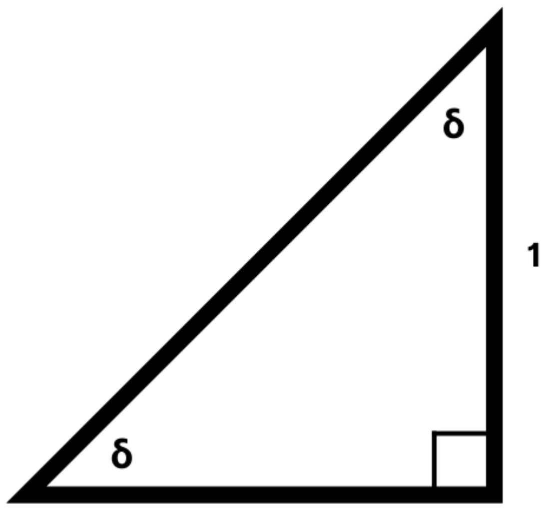
$$\sin^2 55 + \sin^2 35$$

1.3 Trig functions of special angles

1. Using the equilateral triangle below, derive the relations for the 30°60°90°triangle.



2. Use the triangle below to derive the relations of a 45°45°triangle.



3. Find the values of $\sin\frac{\pi}{4}$, $\csc\frac{\pi}{3}$, $\sec\frac{\pi}{6}$, $\cot\frac{\pi}{6}$, $\tan\frac{\pi}{4}$.

4. Find $3 + 4\sin\frac{\pi}{3} - 5\cot\frac{\pi}{6}$

5. Convert 7°15' to degrees.

6. Convert 30' 28" to degrees.

1.4 Trig functions of general angles

List the reference angle (in radians and degrees) of the following:

1. $\theta = 50$

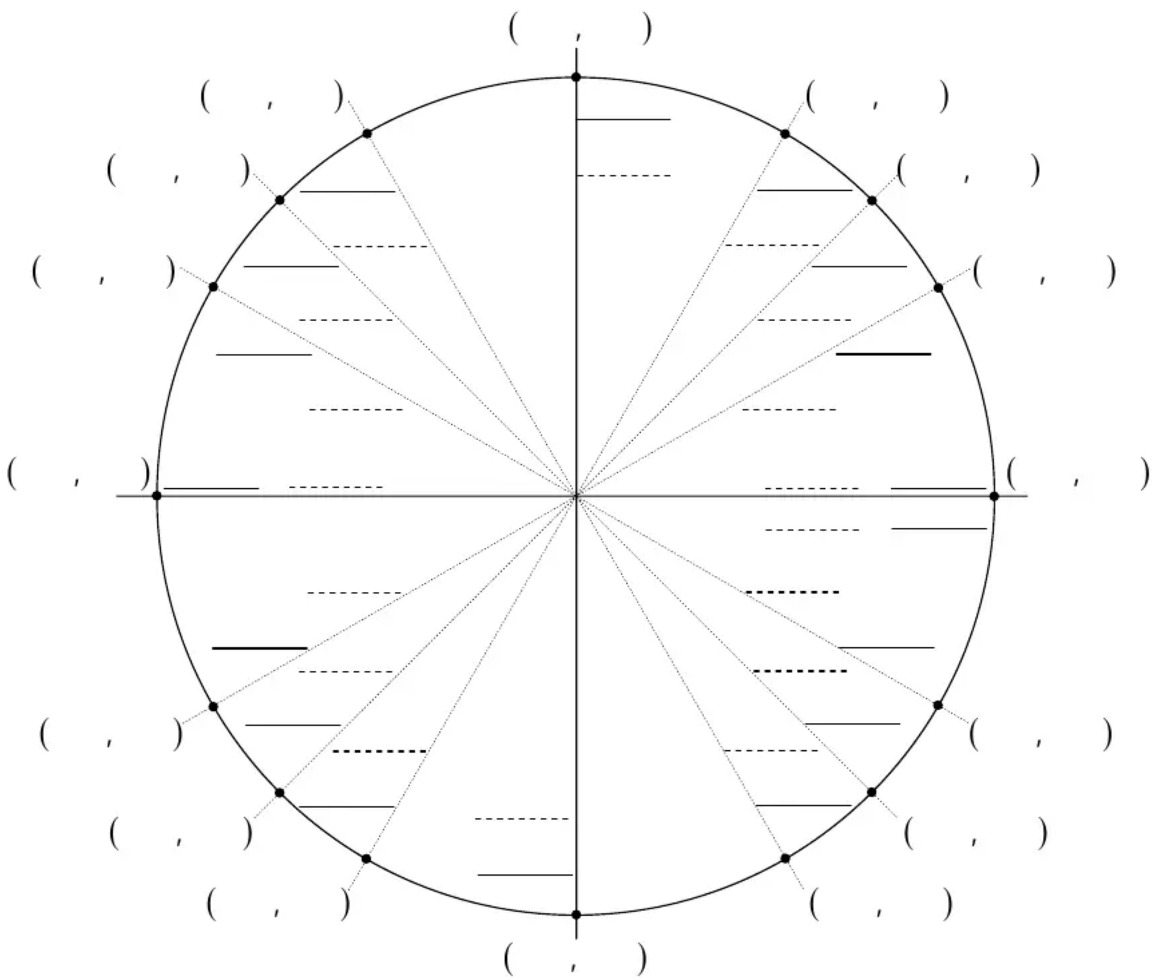
2. $\gamma = \frac{5\pi}{6}$

3. $\rho = \frac{7\pi}{3}$

4. T or F: $\cos(\frac{\pi}{3}) = \sin(\frac{\pi}{6})$

5. T or F: $\sin(\frac{5\pi}{2}) = \sin(\frac{\pi}{2})$

2.1 Describing the unit circle



Locating points

1. For the given number, find the point $P(t) = (\cos(t), \sin(t))$ on the unit circle and the values of the coordinates.

a) $\frac{7\pi}{6}$

b) $-\frac{\pi}{2}$

c) $\frac{13\pi}{6}$

d) 2π

e) π

f) $\frac{\pi}{2}$

g) $-\frac{\pi}{2}$

h) $\frac{4\pi}{3}$

i) $-\frac{4\pi}{3}$

Periodicity

2. Again for the given number, find the point $P(t) = (\cos(t), \sin(t))$ on the unit circle and the values of the coordinates.

a) $\frac{31\pi}{4}$

b) $-\frac{27\pi}{6}$

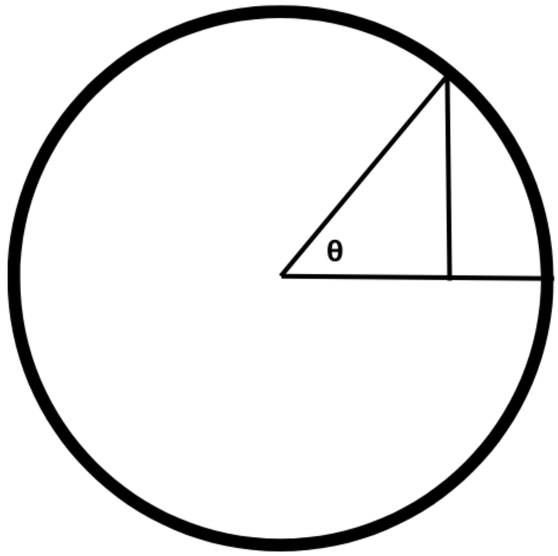
c) $\frac{17\pi}{2}$

d) -31π

e) $18,000 \times 0\pi$

2.2 Circular functions

1. Use the visual aid to fill in θ, $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\csc(\theta)$, $\sec(\theta)$, $\cot(\theta)$.



2. State the domain and range of each of these functions

a) $\sin(\theta)$

b) $\cos(\theta)$

c) $\tan(\theta)$

d) $\csc(\theta)$

e) $\sec(\theta)$

f) $\cot(\theta)$

3. What is an even function?

4. What is an odd function?

5. What is the $\cos(-\frac{\pi}{3})$?

How is it related to $\cos(\frac{\pi}{3})$?

6. What is the $\sin(-\frac{\pi}{3})$?

How is it related to $\sin(\frac{\pi}{3})$?

7. Verify the following:

a) $\cos(x-\pi) = -\cos(x)$

b) $\sin(x-\pi) = -\sin(x)$

a) $\cos(x+\pi) = -\cos(x)$

b) $\sin(x+\pi) = -\sin(x)$

c) $\cos(\pi-x) = -\cos(x)$

d) $\sin(\pi - x) = \sin(x)$

8. Find 3 solutions to the equation $\sin(x) = \cos(x)$, one of them being negative.

9. Find 3 solutions to the equation $\tan(x) = \cot(x)$, one of them being negative.

10. Let S be the sum of solutions of $\sec(x) = \cos(x)$ on $[-3\pi, 3\pi]$. What is $\lceil 9S \rceil$?

11. What is $\sin\left(\frac{7\pi}{12}\right)$?

12. What is $\cos\left(\frac{7\pi}{12}\right)$?

13. What is $\tan\left(\frac{-3\pi}{18}\right)$?

2.3 Graphs of sine and cosine functions

1. Graph the $\sin(\beta)$ function.

2. Graph the $\cos(\beta)$ function.

3. Graph $\sin\left(\beta + \frac{\pi}{2}\right)$

4. Graph $\cos\left(\beta + \frac{\pi}{2}\right)$

5. Graph $\sin(\beta - \pi)$

6. Graph $\cos(\beta - \pi)$

2.4 Graphs of other trig functions

1. Graph the $\tan(\beta)$ function.

2. Graph the $\csc(\beta)$ function.

3. Graph the $\sec(\beta)$ function.

4. Graph the $\csc(\beta)$ function.

5. Graph $\tan\left(\beta + \frac{\pi}{2}\right)$

6. Graph $\csc\left(\beta + \frac{\pi}{2}\right)$

7. Graph $\sec\left(\beta + \frac{\pi}{2}\right)$

8. Graph $\csc\left(\beta + \frac{\pi}{2}\right)$

9. Graph $\tan(\beta - \pi)$

10. Graph $\csc(\beta - \pi)$

11. Graph $\sec(\beta - \pi)$

12. Graph $\csc(\beta - \pi)$

2.5 Special identities

Half-angle identities

1. What is $\sin(\frac{9\pi}{8})$?

2. What is $\cos(\frac{9\pi}{8})$? [*Hint, how could you turn this into a half-angle?*]

Trig substitution

1. How can we rewrite $\sqrt{a^2 - y^2}$ as a trigonometric expression without radicals? [*Hint: draw a triangle and label y and θ*]

2. How can we rewrite $\sqrt{z^2 - a^2}$ as a trigonometric expression without radicals? [*Hint: draw a triangle and what is an identity for* $\sec\theta$*?*]

3. How can we rewrite $\sqrt{y^2 + a^2}$ as a trigonometric expression without radicals? [*Hint: draw a triangle and what is an identity for* $\tan\theta$*?*]

4. How can we rewrite $\sqrt{a^2 + y^2}$ as a trigonometric expression without radicals?

2.6 Inverse trig functions

1. Let $\arccos(\frac{1}{2}) = \frac{t}{n}\pi$, where $\frac{t}{n}$ is in lowest terms. How many prime factors are there of the remainder of $\frac{t}{n}$?

2. Let $\arcsin(\frac{\sqrt{3}}{2}) = \frac{p}{q}\pi$ on $[-2\pi, -4\pi]$, where $\frac{p}{q}$ is in lowest terms. What is the largest prime factor of $|p + q|$?

3. Let $\arctan(-1) = a$ and $\text{arccsc}(-1) = b$ on $[-\pi, \pi]$, where $|a + b| = \frac{w}{z}\pi$. Let w_p be the number of prime factors in w, and let z_p be the number of prime factors in z. What is $w_p + z_p$?

2.7 Trigonometric equations

3.1 Solving right triangles

3.2 Applications of right triangles

3.3 Law of sines

1. $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$

3.4 Law of cosines

2. $c^2 = a^2 + b^2 - 2\,a\,b\cos(C)$

3.5 Simple harmonic motion

Preliminary to complex numbers

1. What is Descarte's rule of signs?

4.1 Complex numbers

1. Solve the equation $x^2 + 4x + 5 = 0$

2. Solve the equation $x^2 + 2x + 4 = 0$

3. How do we define the imaginary unit i?

4. What is $\sqrt{-a}$?

5. What is $\sqrt{-16}$?

6. What is $\sqrt{-20}$?

7. Solve $x^2 + 4 = 0$

8. What are two examples of a complex number?

9. What is the standard form of a complex number?

10. When are two complex numbers equal to one another?

11. How do we add two complex numbers?

12. What is $(3 + 2i) + (4 - 4i)$?

13. What is $(-1 + 5i) - (-3 + 7i)$?

14. How do we multiply two complex numbers?

15. Evaluate:

a) i^2

b) $(2 + 4i)(1 + 2i)$

c) $(7 - 4i)(-3 + i)$

d) $i(2 + 4i)$

e) i^3

f) $i(3 + 4i)(-i)$

g) i^4

h) $i(2i + 3i^4)$

i) i^5

j) i^6

j) What is $i, i^2, i^3, i^4, i^5, ...$?

12) Evaluate

a) $\frac{a+bi}{c+di}$

b $\frac{2+3i}{4-2i}$

4.2 Complex solutions

1. What do you think about the statement:

Given $f(x)$, where $f(x)$ is a polynomial of degree n, and $n > 0$, f has at least one zero in the complex number system.

2. What is the fundamental theorem of algebra?

3. State the degree of the polynomial, the number of roots you expect, find the roots of the function, and plot the functions on the same graph:

a) $f(x) = x - 3$

b) $f(x) = x^2 + x - 2$

c) $f(x) = x^2 + 1$

d) $f(x) = x^3 + 3x^2 - 4x - 12$

e) $f(x) = x^3 + 9x$. How many solutions are there?

f) $f(x) = x^4 - 10x^2 + 9$. How many solutions are there?

g) $x^4 - 1 = 0$. How many solutions are there?

4. Discuss the idea of conjugate pairs if the coefficients of the function are real.

5. Find a fourth degree polynomial with real coefficients that has roots $4i, 1, 1$.

6. Given that $(2 + i)$ is a root of a quadratic, find the quadratic. Does this solve one of our earlier problems?

4.3 Trigonometric form of complex numbers

1. How do you represent the number $2 + i$ in the complex plane?

2. What is the distance from the origin to the point $(2, i)$ in the complex plane?

3. $z = a + bi$. How do you express the absolute value of a complex number? What does it represent?

4. If $z = 2 + 4i$, what is $|z|$?

5. How could you write the number $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ in trigonometric form, ie $d \cos\theta + c\, i \sin\theta$?

6. Write the number $z = -2 - 2\sqrt{3}i$ in trigonometric form.

7. Write the complex number $\sqrt{8}\,[\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})\,]$ in standard form.

8. If $z_a = r_a(\cos(\theta_a) + i\sin(\theta_a))$ and $z_b = r_b(\cos(\theta_b) + i\sin(\theta_b))$, what is $z_a\, z_b$? What is $\frac{z_a}{z_b}$? Feel free

to derive or do research.

9. If $z_a = 3[\cos(\frac{11\pi}{6}) + i\sin(\frac{11\pi}{6})]$, and $z_b = 4[\cos(\frac{5\pi}{3}) + i\sin(\frac{5\pi}{3})]$, what is $z_a\, z_b$? What is $\frac{z_a}{z_b}$?

4.4 Euler's formula

1. What is Euler's formula?

2. What is $e^{i\frac{\pi}{4}}$ using Euler's formula?

4.5 Exponential form of complex numbers

1. If $z = 2(\frac{\sqrt{3}}{2} + \frac{i}{2})$, how could we write this in exponential form? [*Hint, euler's*]

2. What is the exponential form of a complex number?

3. How do $a + bi$, the exponential form, and Euler's formula relate?

4. Express z $= 7\cos(\frac{4\pi}{3}) + i\,7\sin(\frac{4\pi}{3}))$ using all 3 forms. 5. Express z $= 7\sqrt{3}\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4}))$ using all 3 forms.

4.6 DeMoivre's theorem

1. What is DeMoivre's theorem?

2. Use Demoivre's theorem to find $(\frac{1}{2} + \frac{i}{2})$

3. Use Demoivre's theorem to find $(\frac{1}{2} + \frac{i}{2})^2$

4. Use Demoivre's theorem to find $(-1 + i\sqrt{3})^{15}$

4.7 Roots of complex numbers

5.1 Exponential functions

1. The population of the U.S. in 1990 was about 248,718,000. It was predicted to grow at a rate of 8% per decade. Predict the population in 2020 and 2030. Was the prediction for 2020 correct?

2. What is the equation for an exponential function?

3. Given the exponential function formula, which expression represents exponential growth, and which represents exponential decay?

4. What is the domain and range of an exponential function?

Transformations of exponential functions

1. If $f(x) = 3^x$, graph $f(x)$ and $f(-x)$.

2. If $g(x) = 4^x$, graph $g(x)$ and $g(-x)$. What do you notice?

3. If $f(x) = 2^x$, graph three points by hand, indicate the stretch and y-intercept on each, and then check your answer with a computer:

a) $f(x)$

b) $2\,f(x)$

c) $3\,f(x)$

d) $\frac{1}{2}\,f(x)$

e) $\frac{1}{3}\,f(x)$

f) $-2\,f(x)$

g) $-3\,f(x)$

h) $-\frac{1}{2}\,f(x)$

i) $-\frac{1}{3}\,f(x)$

j) $f(x)+1$

k) $2\,f(x)-1$

l) $\frac{1}{2}\,f(x)+1$

m) $\frac{1}{3}\,f(x)-1$

n) $f(x+1)$

o) $2\,f(x+1)$

p) $3\,f(x+1)$

q) $\frac{1}{2}\,f(x+1)$

r) $\frac{1}{3}\,f(x+1)$

s) $-2\,f(x+1)$

t) $-3\,f(x+1)$

u) $-\frac{1}{2}\,f(x-1)$

v) $-\frac{1}{3}\,f(x-1)$

w) $f(x-1)+1$

x) $2\,f(x-1)-1$

y) $\frac{1}{2}\,f(x-1)+1$

z) $\frac{1}{3}\,f(x-1)-1$

4. If $g(x) = \frac{1}{2}^{x}$, graph three points by hand, indicate the stretch and y-intercept on each, and then check your answer with a computer:

a) $g(x)$

b) $2\,g(x)$

c) $3\,g(x)$

d) $\frac{1}{2}\,g(x)$

e) $\frac{1}{3}\,g(x)$

f) $-2\,g(x)$

g) $-3\,g(x)$

h) $-\frac{1}{2}\,g(x)$

i) $-\frac{1}{3}\,g(x)$

j) $g(x)+1$

k) $2\,g(x)-1$

l) $\frac{1}{2}\,g(x)+1$

m) $\frac{1}{3}\,g(x)-1$

n) $g(x+1)$

o) $2\,g(x+1)$

p) $3\,g(x+1)$

q) $\frac{1}{2}\,g(x+1)$

r) $\frac{1}{3}\,g(x+1)$

s) $-2\,g(x+1)$

t) $-3\,g(x+1)$

u) $-\frac{1}{2}\,g(x-1)$

v) $-\frac{1}{3}\,g(x-1)$

w) $g(x-1)+1$

x) $2\,g(x-1)-1$

y) $\frac{1}{2}\,g(x-1)+1$

z) $\frac{1}{3}\,g(x-1)-1$

Natural exponential

1. What does the expression $(1+\frac{1}{x})^x$ approach as $x \to \infty$?

2. Try to think of a function whose slope is always equal to the function. Try some test values.

5.2 Logarithmic functions

1. What is the definition of a logarithmic function?

2. Use the definition of a logarithm to evaluate the following:

a) $\log_3(9) = y$

b) $\log_3(27) = y$

c) $\log_4(1) = y$

d) $\log_{16}(4) = y$

e) $\log_{25}(5) = y$

f) $\log_{64}(2) = y$

g) $\log_{10}(10) = y$

h) $\log_{10}(100) = y$

i) $\log_{10}(7) = y$

h) $\log_{10}(x) = -1$

j) $\log_{27}(x) = -\frac{1}{3}$

k) $\log_{-27}(x) = -\frac{1}{3}$

l) $\log_{10}(-1) = y$

3. Why is $\log_{10}(x) = 0$ when $x = 1$?

4. How could you write the number 4 in the base power of 10?

5. How could you write the number q in the base power of p?

5.3 Exponential and logarithmic equations

Exponential equations

1. Graph the function $f(x) = e^x$. Name some of its properties.

2. Graph:

a) $1000 \cdot e^x$

b) $0.05 \cdot e^x$

c) e^x and e^{2x} on the same graph, label 4 points, and draw the arrows for the shifts.

d) $e^{1/3\,x}$

e) $e^x + \pi$

f) $e^{1/3\,x} - \frac{\pi}{4}$

g) e^{x+1}

h) $e^{\theta - 2\pi}$

Logarithmic equations

1. Graph the function $\log_{10}(x)$

2. Graph the function $\log_{10}(x), \log_9(x), \log_8(x)$ on the same graph. What do you notice?

3. Graph the function $\log_{10}(x), \log_{11}(x), \log_{12}(x)$ on the same graph. What do you notice?

4. Graph the function $\log_{10}(x+1), \log_{10}(x-1), \log_{10}(x)+1, \log_{10}(x)-1$, on the same graph. What do you notice?

5. Graph the function $\log_{10}(x)$, $0.5\log_{10}(x)$, and $2\log_{10}(x)$ on the same graph. Draw the arrows for the shifts.

6. Create two examples of your own for a logarithmic function.

5.4 Exponential and logarithmic models

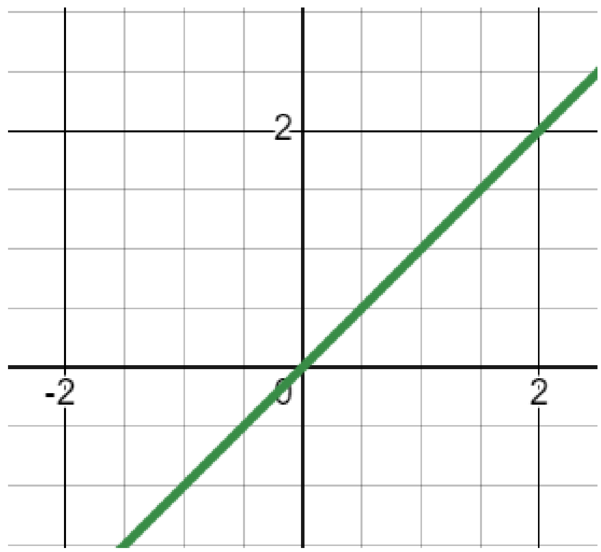
5.5 Hyperbolic functions

Analytic Geometry

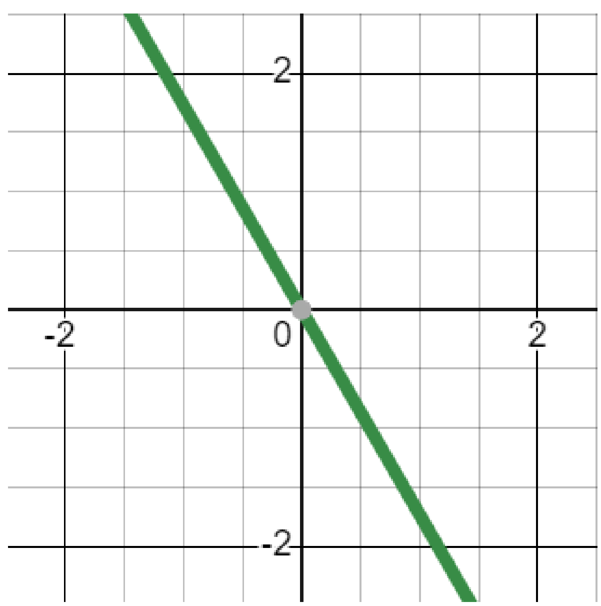
6.1 Lines

1. What is the angle of inclination of a line?

2. Draw the angle of inclination on the graph below and state it in degrees and radians.

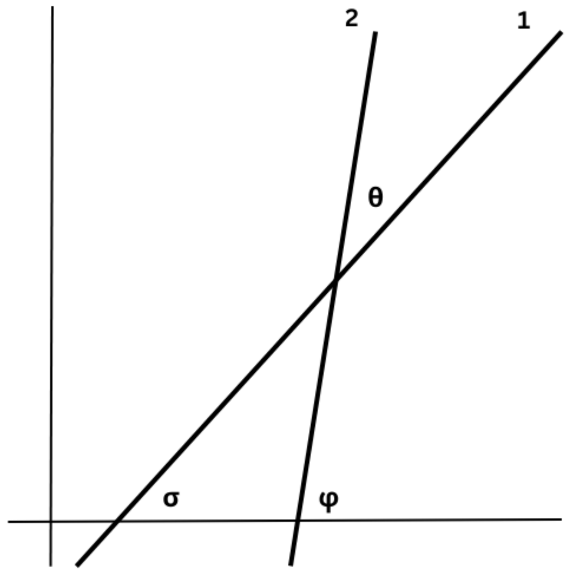


3. Draw the angle of inclination on the graph below and state it in degrees and radians. *Hint, the equation is* $y = -\sqrt{3}\,x$



3. What is the relation between m and $\tan(\theta)$?

4. Find the inclination of the line $2x + 3y = 4$. Give your answer in radians and degrees.



5. State the $\tan(\theta)$ of the angle between the two lines. State in two different forms.

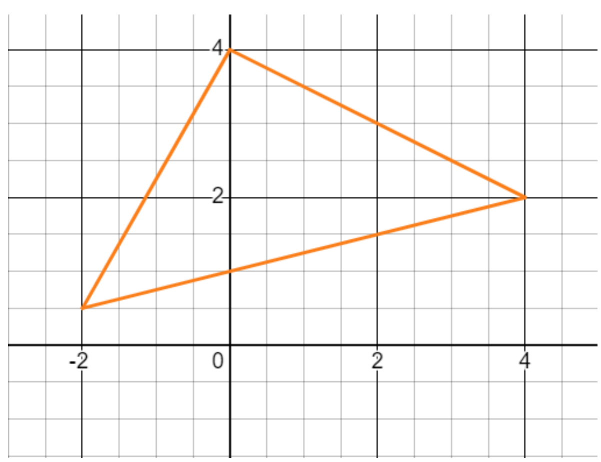
6. a) Find the angle between the two lines *Line 1:* 3x + 4y - 12 = 0 and *Line 2:* 2x - y - 4 = 0. Give your answer in radians and degrees.

b) Graph the two lines and the angle between them.

7. What is the equation for the distance between a point (x_1, y_1) and a line $Ax + By + c = 0$?

8. a) Find the distance between the line $y = 3x + 1$ and the point $(4, 2)$.

b) Graph the line and point.



9. Given the triangle, find a) the altitude of the triangle.

b) The area of the triangle. (*I got a whole number answer. Check this!*)

6.2 Conics

1. List the four basic conics.

2. List the three degenerate conics.

3. What is the general second-degree equation for a conic?

4. What is a locus?

6.3 Parabolas

1. What is the standard form of the equation for a parabola along the vertical axis? State focus and directrix.

2. What is the standard form of the equation for a parabola along the horizontal axis? State focus and directrix.

3. What is equation for a parabola with focus (4,3) and directrix y = -3?

4. What is the equation for a parabola with vertex at the origin and focus (4,0)?

6.4 Ellipses

1. What is the definition of an ellipse? State in your own words.

2. Draw out an ellipse, labeling the 2 foci, a point on the ellipse, the distance from the foci to the point, the vertices, co-vertices, major and minor axes, semi-major and semi-minor axes, with a, b, c, h, k

3. What is the relation between a, b, and c?

4. What is the equation for an ellipse whose major axis is horizontal?

5. What is the equation for an ellipse whose major axis is vertical?

6. Find the standard form of the equation of an ellipse which has foci at $(6, 1), (0, 1)$ and major axis of length 8.

7. What is the eccentricity of an ellipse? What is the eccentricity of the ellipse in problem 6?

6.5 Hyperbolas

1. What is the equation for a hyperbola? How are a, b, and c related in this case?

2. Find the equation for a hyperbola with foci at $(4, 3), (0, 3)$ and vertices $(3, 3), (1, 3)$.

6.6 Rotation of axes

6.7 Parametric equations

7.1 Polar coordinates and equations

1. What are the two parameters for a polar equation?

2. What is the rectangular equation for $r = 3$?

3. What is the equation $\alpha = \frac{\pi}{3}$ in rectangular form?

4. What is the equation $r = \csc \rho$ in rectangular form?

5. What is the equation $r = \sec \eta$ in rectangular form? Is this a function? Is this a relation?

6. What does the equation $r = \theta$ look like?

7.2 Graphs of polar equations

1. True or false:

a) $\sin(-\theta) = \sin(\theta)$

b) $\sin(-\theta) = -\sin(\theta)$

c) $\cos(-\theta) = \cos(\theta)$

d) $\cos(-\theta) = -\cos(\theta)$

e) $\tan(-\theta) = \tan(\theta)$

f) $\tan(-\theta) = -\tan(\theta)$

2. Graph the function $r = 2$

3. Graph the function $\theta = \frac{\pi}{4}$. What is the rectangular equation?

4. For the function $r = \sin(\theta)$, display the relevant unit cirlce values $(0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, ...)$ for r and $\sin(\theta)$ on a polar graph. 5. How do you test for symmetry about the line $\theta = \frac{\pi}{2}$ in a polar system?

6. How do you test for symmetry about the polar axis in a polar system?

7. How do you test for symmetry about the pole in a polar system?

8. Is the curve $r = 2 + 3\cos(\beta)$ symmetric with respect to the polar axis? How do you know?

7.3 Conic sections in polar coordinates

1. How do you define a conic in polar coordinates?